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Symbiotic Fusion Through Single-Crystal Li^6D

Richard L. Liboff[1]

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A symbiotic fusion scheme is introduced. In the first phase of the process, a low-density current of deuterons propagates in a circular tunnel in the interior of a block of single-crystal Li^6D with cubic crystal structure. Energy of the deuterons is limited by the condition that they not be neutralized by collisions with the tunnel wall. Deuteron current is driven through a rapidly rising magnetic field normal to the plane of the loop. Because of the coupling of the beam to the periodic array of molecules in the host, the beam goes into extended states. At the critical temperature ($T_c \approx 9.07$ K) it becomes superconducting and with sufficient wave function overlap in this phase, it is proposed that fusion takes place. A superconducting product wave function is given by Gaussian space components and spin-1 functions polarized in the direction of the applied magnetic field. The Landau–Ginzberg equation is employed to calculate the coherence length for this process which is found to be of the order of the spread of the Gaussian per period of the wave function. This value of coherence length is consistent with significant wavefunction overlap. In the second phase, for a cubic fuel sample of edge length 15 cm, emitted particles interact with the host nuclei in a chain reaction. Assuming a probability of 0.01 that deuterons in the superconducting loop fuse, in 1 ms the device produces a yield ≈ 0.2 GJ. Injection and magnetic rise-time intervals are described and the interval over which the beam goes simultaneously to a superconducting phase from an extended state that fills the current tube. A model is described for this process that gives a criterion for the onset of fusion. A study of deuteron interaction in the superconducting state leads to a requirement for the tensile strength of the tunnel material. Another study derives the property that the Coulomb singularity between deuterons is reduced in the superconducting phase.

KEY WORDS: fusion; wave function overlap; bosons; correlation length; deuterons; superconductivity; symbiotic scheme.

1. INTRODUCTION

Both magnetic and inertial confinement processes for attaining thermonuclear fusion have shown promise in recent years (Janev and Drawin, 1993; Liboff, 1979; Lindl, 1998; Ogansian, 2001) In the present work a noncontinuous process is described on the basis of the following. In the first phase of the process, it is proposed that if a beam of deuterons can be made to enter a superconducting phase then with sufficient wave function overlap, fusion may result. A means of

[1] Schools of Electrical Engineering and Applied Physics, Cornell University, Ithaca, New York 14853.

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achieving this phase is described in terms of a current of deuterons propagating in a circular tunnel within a block of single-crystal Li^6D with cubic crystal structure (fcc). The lattice constant of this material is (4.08 Å) (Ashcroft and Mermin, 1967). The crystal melts at 692°C. Deuteron energies are restricted by the condition that they not be neutralized in collisions with the tunnel wall.

The deuteron current in the tunnel interacts with the periodic potential field of the host crystal and goes into extended states. At a critical low temperature, the deuterons enter a superconducting phase and for sufficiently large wave function overlap in this phase (Ishihara, 1971; Pathria; 1972), it is envisioned that fusion takes place. The superconducting wave function is taken to be a product of single-particle Gaussians and single particle spin-1 states. The Landau–Ginzberg equation (Landau and Ginzberg, 1950) is employed to calculate the coherence length for the system whose value is consistent with significant wave function overlap. Emitted particles then interact with nuclei of the host crystal in a chain reaction that produces the yield. A criterion for the onset of fusion of the system in terms of the extended dimension of a deuteron is described. Three appendices are included. In Appendix A an approximation is made of forces exerted on the confining tunnel due to Coulomb expansion of the beam. In Appendix B an approximation of the interdeuteron force in the superconducting state of the beam is made that is significantly larger than the Coulomb expansion force. In Appendix C, a study of the interaction of a deuteron and its nearest neighbor in the superconducting phase, indicates that at small separation, the Coulomb singularity is diminished. A balance of deuteron density in the confinement tunnel enters the analysis such that if this parameter is too large, the confinment tunnel is unable to support the beam and if it is too small, the fusion yield drops to inefficient values. Three mutually inclusive time intervals enter the analysis: the deuteron-beam injection time, the rise time of the imposed magnetic field, and the operational time of the device.

2. ANALYSIS

In the configuration under study, deuterons in a circular tunnel within a block of single-crystal Li^6D are made to propagate with the application of a rapidly rising magnetic field normal to the plane of the current loop. The beam enters the circular domain though a magnetically shielded cylinder (i.e., material of high magnetic permeability) and connects tangent to the circular tunnel. Consistent with values contained in the relations (4b,6,17), the beam is composed of 1 eV deuterons with current, $I \simeq 110\mu$A. At the circular radius 0.05 m, this current fills the tunnel in the interval $\mathcal{T} \simeq \pi \times 10^{-5}$ s at which point the feed-current cuts off.

As a charged particle in the presence of a dielectric interacts with its image in the dielectric (Jackson, 2001), the current exists in a region with a periodic potential and goes into extended states. Since deuterons are bosons (spin 1), under proper conditions, the current will undergo a transition to a superconducting state (Blatt, 1964; Ibach and Lüth, 1990; London, 1954). Because of the interaction (McMurry and Fay, 1997)

$$\mathrm{Lid} + \mathrm{H_2O} \rightarrow \mathrm{HD} + \mathrm{LiOH}, \tag{1}$$

the device should be operated in a dry room. This crystal does not interact with oxygen at room temperature and below (Lide, 1999).

A rough estimate of this superconducting transition temperature is obtained as follows. We note that the primary microscopic length in the problem is the lattice constant, a. Setting a equal to the thermal deBroglie wavelength (Pathria, 1972) gives the critical temperature

$$T_\mathrm{c} \approx h^2/2\pi Ma^2k_\mathrm{B} \approx 9.07\ \mathrm{K} \tag{2}$$

where M is deuteron mass, k_B is Boltzmann's constant, and h is Planck's constant. At this transition, the current continues with no resistance and any applied magnetic field is expelled from the current loop (Ibach and Lüth, 1990), according to which, the emf of the deuteron current due to a time-changing magnetic field normal to the plane of the ring is given by (Jackson, 2001), (SI)

$$\dot{\Phi} = -\oint \mathbf{E}\cdot d\ell = -2\pi R\rho J \tag{3}$$

where a dot denotes time differentiation, $\mathbf{E}$ is electric field, $d\ell$ is element of arc length of the current loop, Φ is magnetic flux, ρ is electrical resistivity, J is current density, and R is the radius of the current loop, defined as follows. We take the plane of the loop to be the plane through the outer perimeter of the circular tunnel. The intersection of the tunnel with a plane normal to the plane of the loop that divides the tunnel into two congruent sections, gives two circles whose centers are separated by $2R$. The midpoint of this separation is called the center of the loop. If $\mathbf{r}$ is the radial location of a deuteron, measured from the center of the loop, then it is assumed that $r \simeq R$.

If the current tube has cross-section A, the related line current is given by $I = AJ$. Thus,

$$\dot{\Phi} = -2\pi RI\rho/A \simeq B\pi R^2/\tau \tag{4a}$$

where τ is the rise time of the magnetic field, B. Equivalently, we may write

$$|B(W/m^2)| \simeq \frac{2I\rho\tau}{RA} \tag{4b}$$

With the values, $R = 0.05$ m, $A \equiv \pi d^2/4 = 10^{-10}\mathrm{m}^2$, $I \simeq 110\ \mu A$, $\tau \simeq \pi \times 10^{-4}$ s, where d is the diameter of the current beam. Note that this rise time of the magnetic field is larger than the feed time of the deuteron beam by a factor of 10.

Viewing the deuteron current loop as a moderately good conductor, we set (Lide, 1999) $\rho \simeq 10^{-6}\text{ohm} - m$. With these values we obtain

$$|B| \simeq 1.4 \times 10^{-2}(\text{W/m}^2) \tag{4c}$$

We recall $1\ \text{W/m}^2 = 10^4 G$. It follows that

$$|B(G)| \simeq 0.14\ \text{kG} \tag{4d}$$

This estimate implies that a magnetic field over a circular domain of 25 cm^2 that rises to 0.14 kG in 1 ms will produce a current of 110 μA in a circular current loop about the domain. In practice, the magnetic field rises to its peak value and maintains this value in partial confinement of the current loop. In the superconducting phase, the related magnetic flux is a multiple of the fluxoid, $\Phi_0 = 2.0679 \times 10^{-7}$ G - cm^2 (Ashcroft and Mermin, 1967). In a closely allied work it was shown that the ground state of an aggregate of interacting deuterons in a magnetic field has spins polarized in the direction of the field (Liboff, 1994).

We note the deuteron tunnel-wall interaction,

$$\text{LiD} + \text{D}^+ \rightarrow \text{D}_2 + \text{Li}^+, \tag{5}$$

To circumvent this as well as other problems, a thin film of amorphous material inert to hydrogen, with roughly the same dielectric constant ≈ 13.0, of LiD, and ionization energy $\gtrsim$7.7 eV, the ionization energy of LiD (Lide, 1999), coats the interior of the circular tunnel. An additional requirement is that of the tensile strength of the tunnel material required to confine the beam against Coulomb expansion and deuteron interaction, has the value $\gtrsim$0.229 Nt/cm (see Appendix B).

Deuteron energies are limited to values at which they will not be neutralized in collision with the tunnel wall. So deuterons may enter the system at energy $\lesssim$7.0 eV. When the temperature of the system is lowered to the critical value, $\simeq$9.07 K, deuterons thermalize in collisions with the tunnel wall to this temperature. We note that the range of the nuclear force for the deuteron is, $r_\text{D} = 2.3 \times 10^{-13}$ cm. To assist against Coulomb instability of the beam, we stipulate that the mean inter-deuteron displacement is

$$\bar{r}_\text{D} \approx (10^8/2.1) r_\text{D}\text{cm} = 1.1 \times 10^{-5}\text{cm} \tag{6a}$$

which corresponds to the number density

$$n \approx 1/(\bar{r}_\text{D})^3 \approx 7.5 \times 10^{14}\text{cm}^{-3} \tag{6b}$$

The proposed extended state is enhanced by the property that 74% of the probability density of the deuteron lies outside the nuclear core (Blatt and Weisskopf, 1957; Meyerhoff, 1955). For future reference we label this extended displacement, r_D^*.

The Bloch wave function of a particle in the beam is given by

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u(\mathbf{r}) \tag{7}$$

where $u(\mathbf{r})$ is a periodic function with period given by the lattice constant and $\mathbf{k}$ represents "crystal momentum." As noted above, the displacement $\mathbf{r}$ occurs near the radius of the loop. Note that the proposed superconducting state is not the canonical superconductivity of metals involving Cooper pairs, but rather that of a propagating bose fluid (Ishihara, 1971; London, 1954, Pathria, 1972).

3. WAVE FUNCTION OVERLAP

As the configuration has circular symmetry, it suffices to consider deuteron overlap per unit periodic element, that is, per lattice constant of the dielectric. This overlap is given by

$$\Lambda = \frac{1}{\delta} \int dx u_{\alpha}^{*}(x) u_{\beta}(x+a) \tag{8}$$

where α and β are particle numbers, x represents displacement along the current loop, and the integral is over adjacent cells. The parameter $\delta \equiv a/\sigma$, where σ represents spread of the Gaussian. The inverse of δ in (8) incorporates the property that the overlap grows as $\sigma >> a$. With dielectric sites at $x = \pm a/2, \pm 3a/2, \ldots,$ we assume that $u(x)$ is a periodic form of identical Gaussian functions peaked at dielectric sites. With j an odd integer, we write

$$u_j = \sigma^{-1/2} \pi^{-1/4} \exp\left[-\left(\frac{2x - ja}{\sqrt{8}\sigma} \right)^2 \right] \tag{9}$$

for the component of $u_{\alpha}(x)$ centered at $ja/2$. By symmetry, all overlap integrals have the same value and we obtain

$$\Lambda = \frac{1}{\delta} \int_{-a}^{a} dx u_{-1}(x) u_1(x) = \frac{1}{\sigma\delta\sqrt{\pi}} \int_{-a}^{a} \exp\left[-\left(\frac{2x - a}{\sqrt{8}\sigma} \right)^2 \right] \times \exp\left[-\left(\frac{2x + a}{\sqrt{8}\sigma} \right)^2 \right] dx \tag{10a}$$

which reduces to

$$\Lambda = \frac{2}{\delta} \exp[-(\delta/2)^2] \operatorname{erf} \delta \tag{10b}$$

where "erf" denotes the error function (Abramowitz and Stegun, 1965). The overlap has the property that it approaches 2.457 for $\sigma >> a$, $\delta << 1$ and decays exponentially to zero, at $\sigma << a$, $\delta >> 1$ (erf $\delta \approx 1$, for $\delta \gtrsim 1.25$). At $\delta = 0.8$, $\Lambda = 1.58$

A more direct study of the interaction of a deuteron and its nearest neighbor is given in Appendix C, which indicates that in the superconducting phase, at small separation, the Coulomb singularity is diminished.

4. COHERENCE LENGTH

We assume that the superconducting state of our system is given by a product of Gaussian single-particle states (9) and a product of spin-1 states. As was previously noted, it has been shown that the ground state of an aggregate of interacting deuterons in a magnetic field includes all spins in the direction of the imposed magnetic field (Liboff, 1994). Assuming that this property carries over to the present configuration, we take our ground state for an aggregate of N bosons to be given by the symmetric spin-spatial product state

$$\Psi(\mathbf{r}^N, \mathbf{S}^N, t) = \frac{1}{N!}\sum_{\mathbf{P}}\left(\prod_{i=1}^{N}\psi_i(\mathbf{r}_i)\mu_i\right)\ \exp(-i\omega_N t) \tag{11a}$$

$$\hbar\omega_N = E_N \tag{11b}$$

where E_N is the system energy, μ_i represents the spin state of the ith deuteron polarized as noted, and in the sum, $\mathbf{P}$ represents permutation of the N coordinates of the system. In the work of Landau and Ginzberg (1950) and Gross (1958), the ground state of a bose system interacting through a repulsive interaction, was examined employing a Hartree-Fock approach. A nonlinear Schrödinger equation emerges whose solution (Wu, 1961) is uniform except at the boundaries of the system where it vanishes.

A measure of wave function overlap in the superconducting phase is given by the coherence length ξ. The value of this parameter follows from the Landau-Ginzburg equation (Landau and Ginzburg, 1950),

$$\left[\alpha + \beta|\Phi|^2 + \frac{1}{2M}\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\mathbf{A}\right)^2\right]\Phi = 0 \tag{12a}$$

where Φ is a superconducting order parameter with the property that $|\Phi|^2 = n$, the superconducting density. The constants α and β are parameters related to the Landau–Ginzberg analysis relevant to the superconducting phase and A is the vector potential related to the imposed magnetic field. The conherence length ξ is given in terms of α as follows (Marder, 2000; Pischke and Bergersen, 1989; Wu, 1961).

$$\xi^2 = \frac{\hbar^2}{4M|\alpha|} \tag{12b}$$

In the first approximation of ξ the imposed magnetic field vanished. The corresponding spatially uniform solutions are

$$|\Phi|^2 = \Phi_0^2 \equiv -\alpha/\beta,\ \text{or}\ \Phi_0 = 0 \tag{12c}$$

With $\beta > 0$, the real solution Φ_0 corresponds to $\alpha < 0$ and we set

$$\gamma \equiv \frac{\Phi}{\Phi_0} \tag{12d}$$

The Landau-Ginzburg equation (12a) (with zero vector potential) together with (12c) gives the following relation (in one dimension).

$$-\xi^2\gamma'' - \gamma + \gamma^3 = 0 \tag{12e}$$

where primes denote differentiation. We may approximate the coherence length ξ, working with the component wave function (11). Choosing $j = 1$, deleting subscripts and following (12d) we set,

$$\tilde{u} \equiv u/u_0 = \exp(-\phi^2); \qquad \phi \equiv \frac{2x - a}{9\omega} \tag{13a}$$

where $u_0 = u(\phi = 0)$ and x is displacement at about the radius R. Note the values,

$$\phi'(x) = \frac{2}{9\sigma}, \quad \phi''(x) = 0, \quad \phi(0) = -\frac{a}{9\sigma} \tag{13b}$$

Without loss in generality we evaluate the elements of (12e) at the site, $x = 0$. There results

$$2\xi^2 = \left(\frac{9\sigma}{2}\right)^2 \frac{e^{-\phi^2}(1 - e^{-2\phi^2})}{1 - 2\phi^2} \tag{13c}$$

where $\phi = \phi(0) = -a/9\sigma = -\delta/9$. Recalling the estimate in (10), we set $\delta \approx 0.8$. Inserting this value in (13c) we find, $\xi \approx 0.566\sigma$. It follows that for the wave function (11) the coherence length is approximately equal to the spread of the Gaussian per periodic element of the wavefunction (11). We conclude that there is significant overlap of this wave function in the superconducting current loop. We take notice also of the fact that the coherence length, $\xi \simeq 0.566\sigma \simeq 0.45a \simeq 2$ Å$<<$ d, the interior diameter of the superconducting current tunnel.

5. FUSION REACTIONS

The fusion equations relevant to this study, together with energy release and approximate cross-sections (in barns, $1b = 10^{-24}\text{cm}^2$) (Gladstone and Lovberg, 1960; Jarmie and Segrave, 1957) at characteristic energies, are given by

$$d + d \to He^3(0.82) + n(2.45) \tag{14a}$$

$$d + d \to t(1.01) + p(3.02) \tag{14b}$$

$$5\mu b, \quad p + d \to He^3 + \gamma + 5.5\,\text{MeV} \tag{14c}$$

$$7\mu b, \quad p + t \to \alpha + \gamma + 19.8\,\text{MeV} \tag{14d}$$

where α, d, n, and t represents an alpha particle, deuteron, neutron, and triton, respectively, and parenthetical numbers are energies in MeV. Cross-sections of the first two interactions are $\sigma \approx 0.5b$ (Gladstone and Lovberg, 1960; Jarmie and Segrave, 1957) whereas cross-sections of the latter two interactions are too small at given energies and may be disregarded. The processes (14a,b) occur with equal probability. We note that the particles that enter the host are $n, \ p, \ t, \alpha$, and He^3. Reactions between these particles and the host as well as other emitted particles are (Gladstone and Lovberg, 1960; Jarmie and Segrave, 1957)

$$4\,\text{mb}, \quad d + He^3 \rightarrow \alpha(3.6) + p(14.7) \tag{15a}$$

$$d + t \rightarrow \alpha(3.5) + n(14.1) \tag{15b}$$

$$Li^6 + n \rightarrow \alpha + t + 4.6\,\text{MeV} \tag{15c}$$

$$5\,\text{mb}, \quad Li^6 + d \rightarrow [Be^8] \rightarrow 2\alpha(22.4) \tag{15d}$$

$$5\,\text{mb}, \quad Li^6 + t \rightarrow \alpha + He^5 + 15.15\,\text{MeV} \tag{15e}$$

The cross-section for the reaction (15b) is $\sigma \approx 5b$. In (15c), the cross-section is proportional to the reciprocal of the interparticle velocity. However, a resonant neutron absorption occurs at 2.5 MeV with cross-section $\sigma \approx 3.0b$ and a spread of ± 0.5 MeV (Brune and Schmidt, 1974; McLane *et al.*, 1988). This resonance matches well with the 2.45 MeV neutron release in (14a). (A note of caution regarding notation: In the preceding relations, n represents a neutron. In the remaining analysis, n represents deuteron number density in the confinement tunnel.)

6. FUSION MECHANISM

At the critical temperature, T_c, the deuteron current undergoes a transition to a superconducting state which, at the given current density, represents the ground state of the system. Because of large wave function overlap of the system, one may assume that the mean inter-deuteron displacement obeys the relation

$$\langle \Psi(\mathbf{r}^N, \mathbf{S}^N)|(\mathbf{r}_i - \mathbf{r}_j)|\Psi(\mathbf{r}^N, \mathbf{S}^N)\rangle \equiv \bar{r}_{ij} \lesssim r_{\text{D}}^* \tag{16a}$$

where $(\mathbf{r_i}, \mathbf{r_j})$ are deuteron radii, respectively. [The displacement r_{D}^* is defined beneath (6b).] The Hamiltonian of the system may be written

$$H = H_1 + \gamma H_2 \tag{16b}$$

The term H_1 includes the kinetic energies and vector-potential terms of the deuterons and their Coulomb and lattice interaction energies. Relevant to reaction (14a), H_2 includes terms representing annihilation of deuteron pairs and creation of corresponding He_3 and neutron pairs as well as kinetic and interaction energies of these created particles, where both H_1 and H_2 are in second-quantization representations (Akhiezer and Berestesky, 1982; Constantantinescu and Magyri, 1971). The parameter γ is such that for

$$\bar{r}_{ij} > r^*_{\mathrm{D}}, \gamma = 0, \text{ and for } \bar{r}_{ij} \approx r^*_{\mathrm{D}}, \gamma \approx 1 \tag{16c}$$

This description may also be given in terms of energy levels of the system that include a lower-lying level than the ground-state superconducting level. Namely, this lower energy level occurs through the interaction (14a). In this (relativistic) picture, energy is released when the system "falls" to the lower energy state of the fused particles (Bjorken and Drell, 1964).

7. START-UP REACTIONS

Our fusion process begins with reactions in the superconducting loop. Having discovered significant wave function overlap in the ring, we calculate the yield on the basis of this property. Again consider a circle of radius $R = 5\,\text{cm}$. With number-density given by (6b), and the $d - d$ fusion release, $f \approx 3.0\,\text{MeV}$, we obtain the yield

$$Y = nA2\pi Rf(\text{MeV}) \approx 10\,(\text{GeV}) \approx 10^{10}\,\text{eV} \tag{17a}$$

where $A \approx 10^{-10}\,\text{m}^2$, is the cross-sectional area of the tunnel in which the deuteron current propagates. In this expression we assume that the deuteron current is a lineal system in which fusion is due to the number of adjacent pairs in the aggregate. At $v \approx 0.97 \times 10^4$ m/s (corresponding to 1 eV particles), $n = 7.5 \times 10^{20}\ \text{m}^{-3}$, $e = 1.602 \times 10^{-19}$ C the current in the beam is given by

$$I_d = Anev = 116\,\mu\text{A} \tag{17b}$$

This value of deuteron current is consistent with values described in (3) *et seq.* due to a rising magnetic field. It is noted that the reactions (14) are a catalyst to the proposed fusion process. What is important to this process is that particles are emitted in the interaction, not their energy yield. In this context, we note that the number of decay products emitted by the $d - d$ reaction (14a,b), is proportional to the deuteron density in the confinement tunnel, as may be seen in (17a). It follows that the net yield (19a) will also increase with this deuteron density. However, as noted in Appendix B, the force exerted on the confinement tunnel wall similarly is proportional to this density. Thus we have the balance: If this density is too small, fusion yield drops to inefficient values, whereas if it is too large, the tensile strength of the tunnel wall is inadequate to maintain the beam.

8. SYMBIOTIC SCHEME

In the proposed symbiotic scheme, with start-up conditions initiated, the following particle–particle and particle–lattice interactions occur to varying degree (with equation numbers of preceding reactions in parentheses) (Brune and Schmidt, 1974; Gladstone and Lovberg, 1960; Jarmie and Segrave, 1957).

$$n(14a) + Li^6(\text{lattice}) \rightarrow \alpha + t + 4.6\,(\text{MeV})(15c) \tag{18a}$$

$$t(15b) + d(\text{lattice}) \rightarrow \alpha + n + 17.4\,(\text{MeV})(15b) \tag{18b}$$

$$t(14b \text{ or } 15c) + Li^6(\text{lattice}) \rightarrow \alpha + He^5 + 15.15\,(\text{MeV})(15e) \tag{18c}$$

$$He^3(14a) + d(15a) \rightarrow \alpha + p + 18.3\,(\text{MeV})(15a) \tag{18d}$$

In (18a), the neutron emitted in (14a) combines with a lattice deuteron at $\sigma \approx 2b$ with a release of 4.6 MeV. In (18b), the triton of this reaction interacts with a deuteron of the lattice at $\sigma \approx 5b$ with the energy release, 17.6 MeV contributing to a chain reaction. In addition to these reactions the following He^3 reactions are noted (McLane *et al.*, 1988): $He^3(t, d)He^4$ 43%; $He^3(t; p, n)He^4$ 51%; $He^5(t, p)He^4$ 43%. We list the prevalent reaction,

$$t + He^3 \rightarrow He^4 + p + n + 12.1\,\text{MeV} \tag{18e}$$

At 0.3 MeV, $\sigma \approx 30$ mb (Brune and Schmidt, 1974; McLane *et al.*, 1988; Moak, 1953). Thus, the triton of (14b) may react with He^3 of (14a) in accord with the preceding interaction. The neutron release in the preceding reaction can react with lattice Li^6 as in (18a), again contributing to a chain reaction.

9. YIELD ESTIMATES

We estimate the yield of the reaction (18a,b). In this scheme, the neutron emitted in the process (14a) reacts with the lattice Li^6 nucleus at the resonance 2.5 MeV value. In the event that a neutron does not so interact, it will thermalize and interact with this nucleus with the $\propto 1/v$ cross-section within the sample. Concentrating on the resonant absorption, we find the range of the neutron in this crystal $\Delta \simeq 1500\ \mu$m (Littmark and Ziegler, 1990). The yield of this component chain is given by

$$y = \sigma_{N,Li} n_N v_N n_{Li} \Delta^3 \times 4.3\,\text{MeV}/s \tag{19a}$$

where n_N, n_{Li} are neutron and lithium densities, respectively, and v_N is neutron speed. With $\sigma \simeq 3 \times 10^{-24}$ cm^2, $v_N \simeq 2.17 \times 10^4$ cm/s, $n_N \simeq n = 7.5 \times 10^{16}$ cm^{-3}, $n_{Li} \simeq 4.7 \times 10^{22}$ cm^{-3}, we obtain

$$y \simeq 3.34 \times 10^{16}\ \text{MeV}/s \simeq 0.27\,\text{kW} \tag{19b}$$

In the second component of the chain, (18b) comes into play. The range of a triton in the host crystal is 200 μm. The energy of the neutron in this interaction is $\approx$ 14.1 MeV and the cross-section is $\approx 5b$. Repeating the preceding calculation gives

$$y \simeq 21.6 \times 10^{15}\,\mathrm{MeV}/s \simeq 3.45\,\mathrm{kW} \tag{19c}$$

Note in particular, as typical to a chain reaction, products of one reaction participate in another reaction. Each reaction takes place in a volume of order of the cube of the range of the reacting particle in Li^6D. As derived in (19a,b) typically this yield is of the order $\approx$kW. But as the chain reaction proceeds, a portion of the crystal is consumed. An estimate of the yield is given by the volume of the crystal divided by the cube of the range of one of the charged particles. As a component of neurons do not react, we introduce an effective fusion-cube edge-length $\approx$5 cm and effective particle range $\simeq 1\mu$m. In addition we assume a probability of 0.01 that deuterons in the superconducting loop fuse. With the microvolume yield $\simeq$ kW, we then obtain

$$Y \approx 23\,\mathrm{GW} \tag{20a}$$

If the device operates for 1 ms, the yield is

$$Y \approx 0.2\,\mathrm{GJ} \tag{20b}$$

For efficiency of the proposed process, the deuteron beam must go to a superconducting state simultaneously from an extended state distributed over the whole current loop. Deuterons are injected into the tunnel at temperature in excess of the critical temperature, T_c. When current fills the tunnel at $I = 110\,\mu$A, the fuel line cuts off at the time, $\mathcal{T} \approx \pi \times 10^{-5}$ s. After an extended state is established, the temperature drops to the critical value, T_c, and the beam goes uniformly to the superconducting state. Thus, during the first, say, 20 revolutions, an extended state is established, at which time the temperature drops to the critical value and the current beam becomes superconducting. The current is then consumed in approximately 10 revolutions. The corresponding time interval is obtained from the circular velocity given above (17b) and the stated radius of the ring, 0.05 m,

$$t \simeq 2\pi \times 10^{-4}\,\mathrm{s} \approx 1\,\mathrm{ms} \tag{20c}$$

which is small compared to the risetime of the applied magnetic field. The three characteristic times that enter in this device are:

$$(\text{injection time, } \mathcal{T};\ \text{rise time, } \tau;\ \text{operational time,} t) = \pi(10^{-5}\,\mathrm{s}; 10^{-4}\,\mathrm{s}); 1\,\mathrm{ms} \tag{20d}$$

10. CONCLUSIONS

A symbiotic process for attaining thermonuclear fusion was described in terms of effecting a superconducting current of deuterons with current driven in accord with Faraday's law. It is argued that for sufficiently large wave function overlap in this phase, deuterons will fuse. The host material is a section of single crystal Li^6D with cubic crystal symmetry that includes a circular tunnel for the deuteron current. In the second phase of the fusion scheme, particles emitted from the reacting deuteron current interact with themselves and the Li^6D host crystal. A chain reaction ensues which, for a cubic fuel sample of edge length 15 cm, gives a yield ≈02 GJ in approximately 1 ms. Injection and magnetic risetime intervals are described and the interval over which the beam goes simultaneously to a superconducting phase from an extended state that fills the current tube.

11. APPENDIX A

In this appendix an approximate value is obtained for the tensile strength of tunnel-wall material required to confine the deuteron beam against Coulomb expansion. A rough estimate of the related force may be calculated by assuming a linear current model of the beam. The diameter of the circular current loop is $2R$. Let ρ_L denote related linear charge density given by [recall data above (17b)]

$$\rho_L = \frac{Q}{2\pi R}; \quad Q = 2\pi R\, enA; \quad \rho_L = enA = 12 \times 10^{-9}\,\mathrm{C/m} \tag{A1}$$

Consider that a diameter intersects the circumference of the circular current loop at the point O, at the right of the circumference. A point on the lower semicricle of the current loop is given by the intersection of a secant from O to the point at the angle θ that the secant subtends with the diameter. We refer to this intersection point as the "point θ." An element of charge in the current loop at the point θ is

$$dq = \rho_L 2R\, \cos\theta\, d\theta \tag{A2}$$

The differential of electric field at the point O from the charge element at θ is given by

$$dE = \frac{\rho_L 2R\, \cos\theta\, d\theta}{4\pi\epsilon_0 (2R\, \cos\theta)^2} \tag{A3}$$

This field is in the direction of the secant from the point θ to O. Because of symmetry of the circle, it suffices to integrate this form over the lower semicircle. In this process the surviving component of electric field is in the direction of the diameter through O.

$$E = \int_0^{\theta_m} \frac{2\rho_L R\, \cos\theta\, d\theta}{4\pi\epsilon_0 (2R\, \cos\theta)^2} = \frac{\rho_L}{8\pi\epsilon_0 R} \int_0^{\theta_m} \frac{d\theta}{\cos\theta} \equiv D\, \ln\left(\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\right)_0^{\theta_m} \tag{A4}$$

where D is the implied parameter. We take the minimum displacement on the circle of charge to be given by the interparticle displacement $n^{-1/3}$ which gives (with $n^{1/3} \simeq 4.3 \times 10^7\,\mathrm{m}^{-1}$ and $R = 0.05\,\mathrm{m}$)

$$\cos\theta_\mathrm{m} = \frac{1}{2Rn^{1/3}} \equiv \epsilon << 1 \tag{A5}$$

It follows that $\sin\theta_\mathrm{m} \simeq 1 - (\epsilon/2)$; $\theta_\mathrm{m} \lesssim \pi/2$. Inserting these values in (A5) gives

$$E \simeq D|\ln(4/\epsilon)| = \frac{\rho L}{8\pi\epsilon_0 R}|\ln(4/\epsilon)| = 8.71 \times \frac{\rho L}{8\pi\epsilon_0 R} \tag{A6}$$

In the preceding, absolute magnitudes are taken because the integral in (A4) is positive. With

$$\frac{\rho L}{8\pi\epsilon_0 R} = 1.18 \times 10^7$$

we obtain

$$E = 12 \times 10^8 V/m(\text{or } Nt/C) \tag{A7a}$$

With (A1) we find that in 1 cm of beam length there is

$$q = \rho_L \times 10^{-2}C = 12.2 \times 10^{-11}C \tag{A7b}$$

amount of charge. With (A7a) this value gives the force per length,

$$f = qE = 1.31 \times 10^{-3}\mathrm{Nt/cm}, \tag{A8}$$

which is a measure of the tensile strength of the tunnel wall required to contain the deuteron beam against Coulomb expansion. In Appendix B, interaction between deuterons in the superconducting state is evaluated that significantly increases this criterion.

APPENDIX B

In this appendix we make a very rough estimate of the mean inter-deuteron force between deuterons in the superconducting state, effects of which must likewise be constrained by the tunnel wall. Motivated by Appendix C, it is assumed that the interpenetration of deuteron charge clouds of the extended state has an harmonic form. Furthermore, for a narrow beam, if a deuteron is repelled by a nearest neighbor, it is soon repelled in the opposite direction by another deuteron. For the inter-deuteron charge-cloud force we write

$$F \approx gx + kx^2 \cdots, g > k > \cdots \tag{B1}$$

where constants represent "spring constants." With (9) we write

$$\langle F\rangle \approx \frac{1}{\sigma\sqrt{\pi}}\int_{-a}^{a} \exp\left[-\left(\frac{2x-a}{\sqrt{8}\sigma}\right)^2\right] kx^2 \exp\left[-\left(\frac{2x+a}{\sqrt{8}\sigma}\right)^2\right] dx$$

$$= \frac{k}{\sigma\sqrt{\pi}} \int_{-a}^{a} \exp 2\left[-\left(-\frac{4x^2+a^2}{8\sigma^2}\right)\right] x^2\,dx = \frac{k}{\sigma\sqrt{\pi}}$$
$$\times \left(\exp\frac{2a^2}{8\sigma^2}\right)\int_{-a}^{a}\exp\left(-\frac{x^2}{2\sigma^2}\right)x^2 dx \qquad (B2)$$

Here it was noted that the first term in (B1) vanishes because of symmetry. The remaining constant, k, has dimensions of Nt/m^2. To perform this integral we introduce the transformation of variables:

$$x^2 = y, \qquad dx = \frac{dy}{2\sqrt{y}}$$

If we label the integral on the right of (B2), I, then with the preceding transformation,

$$I = \frac{1}{2}\int_{-a^2}^{a^2} \exp(-by)\sqrt{y}\,dy, \qquad b \equiv (2\sigma^2)^{-1} \qquad (B3)$$

$$I = \frac{1}{2}\left[-\frac{\sqrt{y}}{b}\exp(-by) + \frac{\sqrt{\pi}}{2b^{3/2}}\,\mathrm{erf}\,(\sqrt{by})\right]_{-a^2}^{a^2} \qquad (B4)$$

where, we recall,

$$\mathrm{erf}\,z = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\,dt$$

In accord with limits described in Section 3, we set, $\sigma = 2a$ which gives, $b = 1/8a^2$. The argument of the erf function is then, $1/\sqrt{82} = 0.3535$, for which erf $(0.3535) = 0.383$. In evaluating I, only the real component is maintained. We obtain

$$I = \frac{8a^3}{2}\left[-\exp\left(-\frac{1}{8}\right) + \frac{\sqrt{\pi}(8)^{1/2}}{2}\mathrm{erf}\left(\frac{1}{\sqrt{8}}\right)\right] = 0.016a^3$$

The exponential factor multiplying the integral in (B2) is very close to unity, so that

$$\langle F\rangle \approx 0.016a^3\left(\frac{k}{\sigma\sqrt{\pi}}\right) \qquad (B5)$$

For the force constant, k we choose the simplest form, $k = e^2 2\sqrt{\pi}/\epsilon_0 a^4$, so that $a^3k/\sigma\sqrt{\pi} = e^2/\epsilon_0 a^2$, and

$$\langle F\rangle = 0.016(e^2/\epsilon_0 a^2) = 0.016\,(17.44\times10^{-9}) = 0.279\times10^{-9}Nt \qquad (B6)$$

To employ this result for tunnel properties, we calculate the mean force per length. This is accomplished by multiplying $\langle F\rangle$ by the deuteron line density in 1 cm.

With(1A) we write $nA = n_L = \rho_L/|e| = 7.5 \times 10^8/\text{cm}$ so that the effective inter-deuteron force per centimeter is given by

$$\langle \mathcal{F} \rangle = n_L \langle F \rangle = 0.220\, Nt/\text{cm} \tag{B7}$$

This result indicates that forces exerted by inter-deuteron interactions exceed forces due to Coulomb expansion effects described in Appendix A by two orders of magnitude.

APPENDIX C

A significant component of this paper is the hypothesis that in the superconducting state, the Coulomb interaction between deuterons is moderated. In this appendix we are concerned with this property in the domain of small a corresponding to small displacement between adjacent deuterons.

The potential-energy interaction integral between deuterons at $x = \pm a/2$ is given by (Jackson, 2001)

$$V = \frac{K}{\sigma^2 \pi} \int_{-C}^{C} \int_{-C}^{C} \exp\left(-\frac{1}{8}\left(\frac{2x' + a}{\sigma_1}\right)^2\right) \exp\left(-\frac{1}{8}\left(\frac{2x - a}{\sigma_1}\right)^2\right) \frac{dx dx'}{|x - x'|} \tag{C1}$$

where K is the SI constant,

$$K \equiv \frac{e^2}{4\pi\epsilon_0}$$

(with dimensions of energy-length) and $C \equiv Wa/4$, $W \geq 8$. These limits correspond to an interval four times the displacement of Gaussians. The factor $(\pi\sigma^2)^{-1}$, in the coefficient in (C1), in the limit of small (σ_1, σ), relates to the product of two delta functions with one particle at $x = -a/2$ and the other at $x = a/2$. In the present study we take the primed particle to be fixed at $x = -a/2$, which is accomplished by taking the limit $\sigma_1 \to 0$

There results

$$\begin{aligned} V &= \frac{K}{\sigma\sqrt{\pi}} \int_{-C}^{C} \int_{-C}^{C} \delta(x') \exp\left(-\frac{1}{8}\left(\frac{2x - a}{\sigma}\right)^2\right) \frac{dx\, dx'}{|x - x'|} \\ &= \frac{K}{\sigma\sqrt{\pi}} \int_{-C}^{C} \exp\left(-\frac{1}{8}\left(\frac{2x - a}{\sigma}\right)^2\right) \frac{dx}{|x|} \end{aligned} \tag{C2}$$

This integral has a pole at $x = 0$. However we note that in the physical configuration, the closest approach of two deuterons is given by the effective diameter of a deuteron, which we label $2b$. Let the integrand in (C2) be labeled $f(x)$. As this function is very nearly even (especially in the domain of small displacement between deuterons), to facilitate calculation, we introduce the closely allied even function

$$F(x) = \frac{1}{2}[f(x) + f(-x)] \tag{C2a}$$

Then for $b > 0$, integrating $F(x)$ over the limits $(-C, -b)$, (b, C) returns the same result which, when added, gives

$$V \simeq \frac{K}{\sigma\sqrt{\pi}} \frac{a}{\sigma^2}(C - b)L(b) \simeq \frac{K}{\sqrt{\pi}} \frac{a}{\sigma^3} CL(b) \simeq \frac{K}{\sqrt{\pi}} \frac{Wa^2}{4\sigma^3} L(b) \tag{C3}$$

as $C >> b$. The factor L(b) $\propto \ln(b)$ serves to remind us that (C3) becomes logarithmically singular as $b \rightarrow 0$. In the present limit, $a << \sigma$ and σ is large, so that (C3) reduces the Coulomb singularity. Thus we find that in the superconducting phase, the inter-deuteron force is decreased. Note that the potential C(3) is harmonic in the displacement a. This property lends consistency to Appendix B in which the harmonic charge-cloud model in the superconducting phase was employed. It has been noted that the Gaussian wave function (9) is a model form. As described previously, in the study of an aggregate of repulsive bosons, (Gross, 1958; Landau and Ginzberg, 1950; Wu, 1960), it was found that the ground state of the system is uniform. With this observation, it is possible that the potential of interaction, (C3) may be further reduced because of an increase in the effective spread of the wave function.

The particle picture emerges in the limit $(\sigma_1, \sigma) \rightarrow 0$. As noted above, in this limit the exponential factors in the integrand (C1) go to respective delta functions representing particles at $\pm a/2$. The Coulomb singularity then occurs in the limit $a \rightarrow 0$, with no modifying factor. In the superconducting state (9), the width of the Gaussian components are always finite, thereby circumventing this unmodified singular behavior.

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